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TECHNICAL REPORT BRL-TR-2861

RANDOM PROCESSES, TURBULENCE AND DISORDERING FIELDS

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OCTOBER 1987

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188
Exp. Date: Jun 30, 1986

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION / AVAILABILITY OF REPORT
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) BRL-TR-2861	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Johns Hopkins University 6b. OFFICE SYMBOL (If applicable) SLCBR-IB-A	7a. NAME OF MONITORING ORGANIZATION Ballistic Research Laboratory
6c. ADDRESS (City, State, and ZIP Code) Baltimore, MD 21218	7b. ADDRESS (City, State, and ZIP Code) Aberdeen Proving Ground, MD 21005-5066
8a. NAME OF FUNDING / SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 61102A; PROJECT NO. 1L161102AH43; TASK NO. 00; WORK UNIT ACCESSION NO. 00

11. TITLE (Include Security Classification)

Random Processes, Turbulence and Disordering Fields

12. PERSONAL AUTHOR(S)

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13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Report	FROM Jan 86 TO Jan 87		

16. SUPPLEMENTARY NOTATION

*Department of Physics and Astronomy, The Johns Hopkins University

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
19	01		Random Processes, fluctuations, inverse powers
21	02		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The statistical theory of random processes is cast into a Lagrangian form. The formalism requires the existence of an arbitrarily weak random stirring force, playing the role of a disordering field. In scale invariant systems the coupling strength of the weak stirring force can be scaled out and it disappears from the theory. The formalism is applied to the turbulent flow of an incompressible fluid; in particular, we derive the Feynman rules by means of which the distribution of of fluctuations around an average flow can be determined. As an illustration, we discuss a stationary flow between two parallel planes. An expansion in inverse powers of the Reynolds number (analogous to a semiclassical expansion in Quantum Field Theory) is outlined. In particular, we compute the vorticity correlation function to leading order in inverse powers of the Reynolds number.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Csaba K. Zoltani	22b. TELEPHONE (Include Area Code) (301) 278-6108 22c. OFFICE SYMBOL SLCBR-IB-A

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

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I. INTRODUCTION

There are many classical, nonlinear systems exhibiting some kind of chaotic behavior. Examples include the turbulent flow of a fluid, usually described by means of the Navier-Stokes equations, and the behavior of liquids, gases or antiferromagnets above the critical point, among others.

In this paper, we propose to reexamine and further develop an approach to the description of such systems, originally proposed by Martin, Siggia and Rose,[1] and reformulated by DeDominicis and Peliti.[2] This approach promises to yield some qualitative insight into problems of the kind outlined above; moreover, if sufficiently developed, it may be combined with numerical methods in order to increase the accuracy of the results or reduce computing time. We use the formulation of ref. 2 and restrict our attention to autonomous systems. From a qualitative point of view, the method consists of the following. One considers a system described by a set of deterministic nonlinear equations governing the behavior of the dynamical variables. In the approach of refs. 1 and 2, the system is then perturbed by a random force. It is then shown that this theory of a "randomly stirred" system is formally equivalent to a quantum field theory: one can define it, for instance, in terms of a suitable path integral, and compute fluctuations induced by the random stirring force by means of familiar and internally consistent methods.

However, the approach as described in refs. 1 and 2, is, to a certain degree, incomplete from a physical point of view. It is not clear that a system has to be randomly stirred in order to become chaotic: for example, a fluid flow can apparently become turbulent under the influence of some perfectly smooth, non-fluctuating external force, such as a constant pressure gradient in a pipe. Although one may argue that in a "real" flow there is always some randomness present (such as thermal fluctuations in the fluid, roughness of the wall of the pipe, etc.), a recent numerical simulation carried out by Deissler[3] seems to indicate that this is not crucial in the development of turbulence and probably in the chaotic behavior of other systems either. Intuitively, one would like to "switch off" the random stirring force in some smooth manner, and make sure that chaotic behavior persists even if the random perturbation is weak. (This, to a certain extent, would

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[1] P. C. Martin, E. D. Siggia and H. A. Rose, "Statistical Dynamics of Classical Systems," Phys. Rev., Vol. A8, pp. 423-440, 1973.

[2] C. DeDominicis and L. Peliti, "Field-Theory, Renormalization and Critical Dynamics Above T_c: Helium, Antiferromagnets and Liquid-Gas Systems," Phys. Rev., Vol. B18, pp. 353-376, 1978.

[3] R. G. Deissler, "Is Navier-Stokes Turbulence Chaotic?," Phys. Fluids., Vol. 29, pp. 1453-1457, 1986.

justify the conjecture expressed by Martin and DeDominicis[4] that at least some important properties of a turbulent flow are independent of the agitation mechanism.) From the technical point of view, the formalism, as it stands, does not readily lend itself to the development of a systematic loop expansion around some non-trivial, average solution represented by a non-trivial saddle point of the path integral, unless the stirring force is kept nonzero, since all propagators are proportional to the correlation function of the stirring force.

Quite clearly, this calls for a more careful examination of the role played by a random, typically Gaussian, stirring force in the theory of nonlinear systems in which we are interested. It turns out that, within the framework of the formalism created by Martin, Siggia, Rose, DeDominicis, and Peliti (MSRDP), loc. cit., one can associate a Hamiltonian system with every autonomous, but possibly dissipative system stirred by a random force. That Hamiltonian system becomes singular, in a sense to be specified later, if from the outset, the stirring force is set equal to zero. In some simple and physically interesting cases, typically, Gaussian stirring of equal strength at every frequency and wave number, the functional integration over canonical momenta can be carried out in a closed form. This establishes a relationship between the Hamiltonian and Lagrangian forms of the path integral, as long as the stirring force is not exactly zero.

The paper is organized as follows. In the next Section we consider some general properties of systems described by the class of equations, of the form $\partial_t X - F(X) = \epsilon f(x,t)$, where f is a Gaussian stirring force and ϵ is a parameter characterizing the strength of its coupling to the system of interest, while all dynamical variables are collectively denoted by X, where X is an element of some vector field. We explicitly exhibit the correspondence between the MSRDP path integral and a Hamiltonian system and derive the Lagrangian form of the path integral which is suitable for the development of a systematic approximation scheme. In Section 3 the formalism is applied to the turbulent flow of a viscous incompressible fluid. In particular, we derive the general form of the Feynman rules necessary for the computation of the non-Gaussian part of the velocity distribution. In Section 4 this formalism is illustrated on a flow which is Poiseuille on the average; in particular, we compute the two-point velocity correlation function in the Gaussian approximation. The results are summarized and discussed in Section 5.

2. GENERAL FORMALISM

For the sake of completeness, we start with a brief review of the procedure of DeDominicis and Peliti (ref. 2). Consider a "classical" system described by an equation of the type

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[4] P. C. Martin and C. DeDominicis, "The Long Distance Behavior of Randomly Stirred Fluids," Prog. Theor. Phys. Supp., No. 64, pp. 108-123, 1978.

$$\partial_t X - F(X) = \epsilon f, \tag{2.1}$$

where f is a Gaussian random force and ϵ is a numerical parameter, which ultimately will be made vanishingly small. The correlation operator of f is denoted by K and the scalar product is then denoted by the symbol $< \quad , \quad >$.

The principal result of DeDominicis and Peliti is that the characteristic functional of the stochastic process defined in the preceding can be written in the form:

$$G[j] = \int DX\ Df\ Det(\partial_t - \delta F/\delta X)\delta(\partial_t X - F(X) - \epsilon f)e^{i<j,X>}e^{-<f,K^{-1}f>/2}. \tag{2.2}$$

Here, DX and Df stand for an appropriately normalized functional measure and j is an arbitrary source function. The cumulants of the distribution are generated by the functional $W[j] = i^{-1}\ln G[j]$. In what follows, we are mainly interested in the cumulants of any given distribution. Hence, it is convenient to define an equivalence class of functionals of the source j by calling two functionals equivalent if their ratio is independent of the source j. Following the tradition of quantum field theory, members of this equivalence class are identified by the conventional equality sign (=). With this, we now have, using a Fourier representation of the delta functional,

$$G[j] = \int DX\ Df\ DP\ Det(\partial_t - \delta F/\delta X)e^{i<j,X>}e^{-<f,K^{-1}f>/2}e^{i<P,(\partial_t X - F - \epsilon f)>}. \tag{2.3}$$

Integration over the stirring force can be readily performed. The result is:

$$G[j] = \int DX\ DP\ e^{i<j,X>}Det(\partial_t - \delta F/\delta X)\exp i\{<P,\partial_t X> - H\}, \tag{2.4}$$

where $H = <P,F> + i\epsilon^2<P,KP>/2$. (We note that the correlation operator is, by necessity, a Hermitean one, thus, $<f,Kg> = <Kf,g>$ for any admissible pair of vectors f and g.) In what follows, one can set the correlation operator, K, proportional to the unit operator without any substantial loss of generality. (Indeed, such a choice of K is desirable from an intuitive point of view: one "stirs" the system by a white noise, i.e., the correlation function of the stirring force is proportional to $\delta(t-t')$ and the stirring is of the same strength at every wave number.) With this simplification, let us now take a look at the functional integral, eq. (2.4). Quite obviously, this integral is of the canonical form (see e.g., Itzykson and Zuber[5]). One is integrating over the phase space of variables X,P, with a weight $\exp i\{<P,\partial_t X> - H\}$. Clearly, the quantity in curly brackets can be identified with the

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[5] C. Itzykson and J. B. Zuber, "Functional Methods," Quantum Field Theory, McGraw-Hill, New York, 1980, Chapter IX, pp. 425-474.

canonical form of the action governing the fluctuations, while H plays the role of the Hamiltonian. The important point to notice is that if one insisted on discarding the coupling between the stirring and the system under consideration, the latter would become a singular Hamiltonian system (in the sense of classical mechanics): one cannot pass freely between the Hamiltonian and Lagrangian formulations of the problem at hand since the second derivative operator, $\delta^2 H/\delta P \delta P$, is singular (in this case, identically zero) when $\epsilon = 0$. (Given the fact that the integration over the canonical momenta, P, is a Gaussian one, reflecting the Gaussian nature of the stirring, there is a one-to-one correspondence between the analysis of the "classical" Hamiltonian system defined implicitly by eqs. (2.3) and (2.4) and the actual system under consideration.)

Let us now accept a unit correlation operator of the stirring force. One can then readily perform the integration over the canonical momenta, thus arriving at the "Lagrangian" form of the functional integral, viz.,

$$G[j] = \int DX e^{i<j,X>} Det(\partial_t - \delta F/\delta X) \exp -\epsilon^{-2} 1/2 <(\partial_t X - F),(\partial_t X - F)>. \qquad (2.5)$$

In general, it is possible to exponentiate the functional determinant appearing in eqs. (2.4) and (2.5) by introducing the usual Fadeev-Popov ghost fields, cf., Itzykson and Zuber, loc. cit.. We denote the ghost fields by u and v; then using the standard expression of a functional determinant in terms of an integral over Grassmann fields, we get the final expression for the generating functional:

$$G[j] = \int DX\, Du\, Dv\, e^{i<j,X>} \exp - \{\epsilon^{-2} 1/2 <(\partial_t X - F),(\partial_t X - F)> - \Gamma\}, \qquad (2.6)$$

where Γ stands for the ghost part of the action, viz.,

$$\Gamma = <u,(\partial_t - \delta F/\delta X)v>. \qquad (2.7)$$

The expression of the characteristic functional obtained in eq. (2.6) and (2.7) has the advantage that it allows one to perform a consistent loop expansion (in powers of ϵ^2) in a standard fashion, cf. Itzykson and Zuber, loc. cit., Ch. 6. In essence, ϵ^2 plays the same role in this formalism as Planck's constant does in quantum field theory. It follows that it is impossible to put $\epsilon = 0$ from the outset: this is a singular limit in the same sense as the limit $h \to 0$ is in quantum theory. In a general system all one can do is to rescale the variables of the functional integration in a manner analogous to a quantum field theory with a single coupling constant; in this way, a "small ϵ" expansion can be converted to a "weak nonlinearity" expansion. A practically important exception to this scaling argument is given by a scale invariant system, notably the hydrodynamics of a viscous incompressible fluid. In this case, a scale transformation can be chosen such that the small parameter ϵ is altogether eliminated from the theory. This case will be studied in the next Section.

We end these general considerations by exhibiting a conservation law intrinsic to the general formalism, provided the system is autonomous, i.e., F is independent of t. Given any autonomous Lagrangian system, the Legendre transform of the Lagrangian with respect to the time derivatives of the dynamical variables (the "energy") is time independent. A straightforward application of this theorem to the Lagrangian appearing in eq. (2.5), viz., $L = 1/2\langle(\partial_t X-F),(\partial_t X-F)\rangle$ leads to the conservation of the quasi-energy

$$Q = 1/2\langle(\partial_t X-F),(\partial_t X+F)\rangle. \tag{2.8}$$

Unfortunately, the quasi-energy is not a positive functional. However, just like the Lagrangian, Q vanishes for an unperturbed ($\epsilon = 0$) solution of the field equations. Therefore, its magnitude characterizes the randomness of the response of the system under the influence of arbitrarily small random perturbations.

3. TURBULENCE IN AN INCOMPRESSIBLE VISCOUS FLUID

We now apply the general formalism outlined in the previous section to the turbulent flow of a viscous, incompressible fluid. The equations governing the flow are:

$$N^i[v] \equiv \partial_t v^i + (v^s\partial_s)v^i + \partial^i p - \nu\nabla^2 v^i = \epsilon f^i, \tag{3.1}$$

$$\partial_i v^i = 0, \tag{3.2}$$

where v^i stand for the components of the velocity, p is the pressure and f is a perturbing Gaussian random force with unit correlation operator. In order to apply the formalism outlined in Sec. 2, we introduce the momentum canonically conjugate to the velocity field, w_i. Then, using eq. (2.4), the action entering the expression of the characteristic functional reads:

$$A = \int d^3x dt\{w_i N^i[v] - 1/2\ \epsilon^2 w^i w_i\}. \tag{3.3}$$

One readily convinces oneself that the part of the action linear in w_i is invariant under the following scale transformation:

$$t = \lambda t';\ x = \lambda^{1/2}x';\ v = \lambda^{-1/2}v';\ p = \lambda^{-1}p';\ w = \lambda^{-1}w', \tag{3.4}$$

where λ is an arbitrary positive scale factor and v, w stand for the magnitudes of the velocity and momentum vectors. (This is just the usual scaling law of the Navier-Stokes equations; in particular, the Reynolds number is an invariant of the transformation.) Therefore, upon choosing $\lambda = \epsilon^{-4}$, the coefficient of the term quadratic in the canonical momentum becomes unity: the small parameter disappeared from the theory altogether. (More precisely, it was shifted into the boundary condi-

tions, which, however, can be adjusted accordingly.) We can now go directly to eq. (2.5); the "Lagrangian" form of the action reads:

$$A = 1/2 \int d^3x dt \; N^i[v] \; N_i[v]. \tag{3.5}$$

This action has to be supplemented by the ghost contribution, Γ, cf., eqs. (2.6) and (2.7). On denoting the ghost fields by ξ and η, respectively, we have:

$$\Gamma = \int dt \; d^3x \{\xi_i(\partial_t - \nu\nabla^2 - v^r\partial_r)\eta^i + \xi_i v^i{}_{,j}\eta^i\}. \tag{3.6}$$

In order to develop a systematic loop expansion to the functional integral with this action, we need to determine an average flow which is a solution, V^i, of the unperturbed Navier-Stokes equations, $N^i[V] = 0$ and of the subsidiary condition, $\partial_i V^i = 0$. The solution to these equations, together with $\xi = \eta = 0$ (which is the only saddle point of Γ), determine a saddle point of the integrand in the functional integral expression of the generating functional. Let us suppose that such a solution has been found. We can then expand the action around the saddle point noting that the fluctuations, $u^i \equiv v^i - V^i$, have to be divergence-free. In order to satisfy the transversality condition, $T \equiv \partial_i u^i = 0$, one has to insert the standard Fadeev-Popov factor, $\delta(T)\cdot \mathrm{Det}(\delta T/\delta u)$, under the functional integral. The determinant is, however, independent of the integration variables, and thus no ghost has to be introduced for this constraint. It is further a straightforward matter to show that the effect of the delta functional is merely to project out the transverse part of the propagator, by means of the projector, $T_{ij} = \delta_{ij} - \nabla^{-2}\partial_i\partial_j$.

After inserting $v^i = V^i + u^i$ into the action and using $N^i[V] = 0$, the action splits up as follows:

$$A = A[V] + A_0[u] + A_1[u], \quad \Gamma = \Gamma_0[\xi,\eta] + \Gamma_1[\xi,\eta,u] \; , \tag{3.7}$$

where the subscripts 0 and 1 stand for the quadratic ("kinetic") and higher order ("interaction") parts of the action describing the fluctuations, respectively. The general expression of the various pieces can be best described as follows. Introduce the following quantities:

$$l_i = (\partial_t - \nu\nabla^2 + V^r\partial_r)u_i + V_{i,r}u^r,$$

$$L_{ik} = (\partial_t - \nu\nabla^2 + V^r\partial_r)\delta_{ik} + V_{i,k}. \tag{3.8}$$

We then have

$$A_0[u] = 1/2\int dt\ d^3x[l_i l^i + \mu^{-2}(\partial_i u^i)^2],$$

$$A_1[u] = 1/2\int dt\ d^3x\{2(l^i u^k u_{i,k}) + (u^i u_{k,i})(u^r u^k{}_{,r})\},$$

$$\Gamma_0 = \int dt\ d^3x \xi^i L_{ik} \eta^k,$$

$$\Gamma_1 = \int dt\ d^3x \xi^i (u^r \partial_r \delta_{ik} + u_{i,k})\eta^k. \qquad (3.9)$$

From here one immediately reads off the general Feynman rules needed for the computation of the non-Gaussian part of the distribution. In particular, the ghost propagator is given by L^{-1} whereas the propagator of the velocity fluctuations is given by $(L^\dagger L)^{-1}$. Clearly, in order to proceed any further, the average flow has to be specified. In the next section, this procedure is illustrated for the important example of a flow which is, on the average, Poiseuille.

4. FLUCTUATIONS IN A FLOW BETWEEN PARALLEL PLANES

We consider a flow taking place between two parallel planes; the pressure gradient is stationary and homogeneous; its direction is parallel to the planes. We assume that the average flow is given by Poiseuille's solution of the Navier-Stokes equations, viz., in a suitably chosen Cartesian system of coordinates, the nonvanishing component of the average velocity is

$$V_z = 1/2\ U(1 - x^2/a^2), \qquad (4.1)$$

where U is related to the pressure gradient and kinematic viscosity in a well-known way, and "a" stands for the half-distance between the planes confining the flow. The coordinate axes x and z are perpendicular and parallel to the planes, respectively, whereas the y-axis is perpendicular to the (x,z) plane. Our objective is to work out the expression for the action with this average flow, and then carry out the inversion of L and $L^\dagger L$ in order to obtain the Feynman rules which ultimately determine the correlation functions. Even with this simple average flow, this is a rather tedious task and the result cannot be obtained in a closed form. In order to proceed, we rescale all quantities in order to measure them in their natural units. To this end, the unit of length is chosen to be a, the unit of time is the characteristic advection time, a/U, and velocities are measured in units of U. In addition, the fluid is characterized by an intrinsic diffusion velocity, $u^* = \nu/a$. The Reynolds number is defined to be $R = U/u^*$. We can choose the units in which the correlation operator of the stirring is measured such that it is proportional to $(u^*)^3 a^2$. In this way, if we go over to dimensionless quantities, the quadratic part of the action, A_0, will be proportional

to R^3. In these units the average flow is obviously given by $V_z = 1/2(1-x^2)$. Next, define, as in the preceding section, $l_i = L_{ik}u^k$, where u^k is the velocity fluctuation in its natural units and L_{ik} is the operator (again using natural units):

$$L_{ik} = (\partial_t - R^{-1}\nabla^2 + V^r\partial_r)\delta_{ik} + V^i,k. \tag{4.2}$$

In this way we get,

$$A_0 = R^3\int dt d^3x l^i l_i. \tag{4.3}$$

It is now obvious that at large Reynolds numbers the action is proportional to the large number, R^3. In the same way as in quantum field theory the semiclassical limit ($h \to 0$) is obtained by rescaling the fields with an appropriate power of Planck's constant; here we remove the large number in front of A_0 by rescaling, $u^i \to R^{-3/2}u^i$. In this way, the quadratic part of the action becomes O(1) in R, whereas the non-Gaussian part carries negative powers of R; thus it is relatively unimportant for large Reynolds numbers. (In fact, one can define a systematic expansion in inverse powers of R; here we restrict ourselves to the leading term of that expansion.) It is also easily seen that in the leading approximation to the distribution, the ghost fields decouple from the velocity fluctuations. Thus, in the leading approximation, one has a Gaussian distribution of fluctuations for $R \gg 1$. In order to compute the correlation function of the fluctuations, we now have to invert the operator $L^\dagger L$. This is best done in a mixed representation: we Fourier transform in the variables t, y, z, denoting the conjugate variables by ω, k_y k_z, respectively. One is tempted to omit the term of $O(R^{-1})$ in the expression of L altogether. However, in this way, the viscous effects would be totally neglected. Instead, at least for the purpose of illustration, we omit the term $R^{-1}d^2/dx^2$ from eq. (4.2), thus making the distribution independent-valued[6] in x. This is expected to be a reasonably good zeroth approximation outside the boundary layer which, in natural units, is approximately of thickness $R^{-1/2}$. In this approximation, the computation of the velocity correlation function (the propagator of the fluctuation) is a straightforward, although somewhat tedious, task: the technical complication arises from the application of the nonlocal projector, T_{ij}, on $(L^\dagger L)^{-1}$. If we are to remain in the leading approximation, however, we do not need the full propagator. The correlation function of vorticities is easier to compute: one just has

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[6] I. M. Gel'fand and N.Ya. Vilenkin, Generalized Functions, Vol. 4, Academic Press, New York, 1964, Chapter III.

to take the curl of a matrix element of the operator $(L^{\dagger}L)^{-1}$ in both of its variables. (Obviously, for an incompressible fluid, the vorticity correlation function contains the same physical information as the full correlation function.) Denoting the components of the vorticity correlation tensor by Ω_{ik}, the result of a straightforward computation can be described as follows.

i) From the reality and symmetry of the vorticity correlation tensor in coordinate space, one readily derives a Hermiticity relationship in the mixed representation used here, viz.,

$$\Omega_{ij}(-k_y,-k_z,-\omega;x,x') = \Omega_{ji}(k_y,k_z,\omega;x',x)^{*}. \tag{4.4}$$

ii) Taking this into account, we list a set of independent vorticity correlation functions as follows.

Let us define the quantity, $B = \{i[\omega+1/2(1-x^2)k_z] + R^{-1}(k_y^2 + k_z^2)\}$.
Then,

$$\Omega_{xx} = -|B|^{-2}[k_y^2(1 + \frac{x^2}{|B|^2}) + k_z^2]\delta(x - x'),$$

$$\Omega_{yy} = \frac{\partial^2}{\partial x^2}\{|B|^{-2}(1 + \frac{x^2}{|B|^2}\delta(x - x'))\}$$

$$- ik_z \frac{\partial}{\partial x}\{xR^{-1}(k_y^2 + k_z^2)|B|^{-4}\delta(x - x')\} - \frac{k_z^2}{|B|^2}\delta(x - x'),$$

$$\Omega_{zz} = - \frac{\partial^2}{\partial x^2}[|B|^{-2}\delta(x - x')] - k_y^2|B|^{-2}\delta(x - x'),$$

$$\Omega_{yx} = - k_y\{i \frac{\partial}{\partial x}[|B|^{-2}(1 + \frac{x^2}{|B|^2})\delta(x - x')] + xk_z \frac{B}{|B|^4}\delta(x - x')\},$$

$$\Omega_{zx} = - ik_z \frac{\partial}{\partial x}[|B|^{-2}\delta(x - x')] + xk_y^2 \frac{B}{|B|^4}\delta(x - x'),$$

$$\Omega_{zy} = k_y\{k_z B^{-2}\delta(x - x') + i \frac{\partial}{\partial x}[\frac{B}{|B|^4}x\delta(x - x')]\}. \tag{4.5}$$

Not surprisingly, the correlation functions are proportional to δ-functions and their derivatives in x: this is to be expected for independent-valued distributions, cf., Gel'fand and Vilenkin, loc. cit. Accordingly, one has to fold the correlation functions quoted with the efficiency functions of the detectors in order to get physically meaningful expressions.

5. DISCUSSION

The formalism developed in this paper allows one to develop a systematic approximation procedure to the theory of stochastic systems originally developed in refs. 1 and 2. In particular, as it was demonstrated in the preceding section, one can obtain useful analytical results in the form of an asymptotic expansion in inverse powers of R in, at least some, turbulent flows. However, the usefulness of this technique goes beyond the large R expansion. In its present form, the formalism appears to be much more amenable to the application of standard, non-perturbative techniques extensively used in quantum field theory, such as a renormalization group analysis of the various correlation functions. The technical reason for this is that the present, Lagrangian formulation of the functional integral is better adapted to the application of such techniques than the essentially canonical formulation in refs. 1 and 2. In those papers, one was forced to use retarded, rather than Feynman propagators: this circumstance alone rendered the formalism so complicated that an efficient implementation of modern techniques was virtually impossible.

In addition, it appears that using the proposed formalism one can gain some interesting insights into the theoretical description of chaotic behavior in general and the theory of turbulence in particular. It has been emphasized in the literature[7] that chaotic behavior has many features which are similar to phase transitions or dynamical symmetry breaking. In this respect, it is particularly pleasing to notice that a consistent statistical description of chaotic phenomena appears to require the presence of an arbitrarily weak but nonvanishing "disordering field," just as a consistent theory of dynamical symmetry breaking requires the presence of a weak "ordering field."[8]

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[7] Hao Bai-Lin, Chaos, World Scientific, Singapore, 1984, Chapter 1.

[8] G. Domokos and P. Suranyi, "Spontaneous Symmetry Breaking in Quantum Field Theory," Sov. Jour. Nuc. Phys., Vol. 2, pp. 361-367, 1966.

ACKNOWLEDGEMENT

In carrying out this research, one of us (G. Domokos) has been supported in part by the Army Research Office, under Contract No. DAAG29-81-D-0100. The second-named author acknowledges partial support by the U.S. Department of Energy, under Grant No. DE-FG01-85ER40211.

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